ABŪ KĀMIL'S
"ON THE PENTAGON AND DECAGON"

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Japanese Studies in the History of Science

Supplement 2

1971

The History of Science Society of Japan

Publication Office
THE HISTORY OF SCIENCE SOCIETY OF JAPAN
Department of Humanities
Tokyo Institute of Technology
2–12–1, O-okayama, Meguro-ku,
Tokyo, Japan

Overseas Distributors
JAPAN PUBLICATIONS TRADING CO.,LTD.
P.O.Box 5030 Tokyo International
Tokyo, Japan
and
1255 Howard Street
San Francisco, Calif. 94103
U.S.A.

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by

Mohammad Yadegari and Martin Levey [1]

Abū Kāmil Shujāᶜ ibn Aslam ibn Muḥammad ibn Shujāᶜ (ca. 850-930 A.D.) was known as al-Ḥasib al-Misrī, "the reckoner from Egypt." He is, after al-Khwārizmī (ca. 825), the earliest algebraist of the Islamic Middle Ages whose writings are extant[2]. His work is important in the history of mathematics for a number of reasons.

He was among the early Muslim algebraists whose work in algebra was extensively used by Europeans. It has been established that Leonardo Fibonacci (of Pisa) had access to the treatises of abū Kāmil. Leonardo was aware of "On the Pentagon and Decagon" of abū Kāmil and used it in his Practica Geometriae. There is proof that Leonardo used dozens of abū Kamil's problems in his algebra[3]. From "On the Pentagon and Decagon", Leonardo used seventeen of its twenty problems carrying over the exact number facts[4].

In previous works, Levey has shown that abū Kāmil was much interested in developing a mathematical methodology which combined the more abstract Greek methods with more pragmatic procedures of the Babylonian and Egyptian algebraists. Evidence for this has been established from his Algebra and his Indeterminate Equations[5]. Further proof is in "On the Pentagon and Decagon" to be discussed.

Up to now there have been several translations of "On the Pentagon and Decagon." Because the Arabic text was lost until Levey discovered it in Istanbul about ten years ago, only the Latin and Hebrew texts were known. The first to be translated was the Hebrew into Italian[6]; then the Latin was carried over into German[7]. The Latin and Hebrew were originally translated from the Arabic of abū Kāmil, contrary to other published statements.

The Arabic text of "On the Pentagon and Decagon" with which we are concerned is to be found in Istanbul, in the Kara Mustafa Library, number 379. It has twenty-one lines to the page and is written in a Naskhi hand; it goes from the title page on fol. 67b to fol. 78b. In this original text, there are no vowels, no commas, or other punctuation. Further, the text is completely rhetorical. This, by the way, is not true for the Hebrew and Latin translations; the former shows only a few notational abbreviations while the Latin shows fractional symbols.

In the text, abū Kāmil does not use words for "plus" and "minus" often. Rather, he uses "and" for "plus" and "except" for "minus." It is interesting that abū Kāmil puts a line segment symbol above each geometric line designation as well as a point. For instance, when he refers to point M, he writes point $\overline{M}$. All of abū Kāmil's geometric figures are constructed precisely. In naming an unknown, abū Kāmil calls it "thing." In the text, the square of the thing is called <u>māl</u>. In the same way <u>māl māl</u> is X^4, <u>māl māl māl</u> is X^6, and <u>māl māl māl māl</u> is X^8.

The book on the pentagon and decagon consists of twenty geometric problems. The treatment, however, of these problems is almost entirely algebraic. This algebraic treatment of geometrical problems may be contrasted with Euclid's geometric treatment of algebraic problems in the <u>Elements</u>. Abū Kāmil's method is closer to that of the Babylonian procedures.

The first ten problems deal with finding the sides of inscribed and circumscribed pentagons, decagons, and fifteen-sided regular polygons when given the diameter of the circle, and vice versa. These problems, arranged progressively, lead a reader to believe that abū Kāmil is truly moving toward some method of approximation, here finding the circumference of a circle by the method of exhaustion. This idea does not materialize in the work. Instead, abū Kāmil devotes himself to the determination of geometric details of his figures. The approximative attempt was not taken up again after abū Kāmil for a long time. The time was not yet ripe to work out anything more general for the ideas of exhaustion approximation.

Abū Kāmil used the generalized formula to obtain the roots of a quadratic equation. In his rhetorical method, in