A History of Entropy through Various Methods: Specially Focused on Technical Term Analysis

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Abstract

Rudolf Clausius's 16 papers on the mechanical theory of heat have been studied through four various methods, i.e. traditional text analysis with the help of Clausius's own manuscripts, mathematical equation analysis, experimental data table analysis, and technical term analysis. The first three analyses were briefly summarized while the result of the last technical analysis was explained with such important terms in thermodynamics as Disgregation (Degree of dispersion) and Uncompensirte Verwandlung (Non compensated transformation). These terms played important roles through indicating the micro nature and irreversible character, respectively before the appearance of the term Entropie (entropy) in Clausius's famous paper of 1865. The result of technical term analysis for his paper on the theory of electricity (1853) by the use of a text mining method is also shown with tables and figures.

Key words: R. Clausius, Entropy, mechanical theory of heat, irreversible (non reversible), text mining

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1. Introduction

We have been studying the papers of Rudolf Clausius (1822–88) published between 1847 and 1873 under the name of the mechanical theory of heat for the past 20 years, and published a book, in which our research papers were mostly collected in 2002.[1] On this occasion the results of our studies through four important analyses will be mentioned.

1.1. Traditional Text Analysis with Clausius's own Manuscripts

We discovered the strong influence of Joseph Fourier's work, the analytical theory of heat on Clausius's mathematical approach through studying Clausius's own manuscript which was in the style of a notebook, called "Aus Wärmetheorie von Fourier (1848)" HS 6452 at the Archive section of the Library of the Deutsches Museum in Munich.[2] It was

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1 Eri Yagi, *A Historical Approach to Entropy, Collected Papers of Eri Yagi and Her Coworkers* (Tokyo: International Publishing Institute, 2002). Eri Yagi, *A Supplement of the Collected Papers of Eri Yagi and Her Coworkers, A Database from R. Clausius's Abhandlungen I–XVI*(Kawagoe: Eri Yagi Institute of Science, 2002).

2 J. Fourier, *Theorie de la chaleur* (Paris. 1822) in *Oeuves de Fourier*, part 1(Paris, 1887): 1–563.

found out that Fourier's mathematical method was well applied by Clausius, e.g. taking the difference between the heat flow of "in and out" for the case of the Carnot cycle's working substance (for example an ideal gas), Clausius succeeded in obtaining the heat (the difference) in the form of the 2nd order differentials. Then he calculated a ratio between the above heat and work (produced), to be a constant A (reciprocal to Joule's "J"). Finally, Clausius's first law of thermodynamics was presented in the form of the 2nd order differentials. Detailed discussions were published in our paper.[3]

1.2. Mathematical Equation Analysis

We created a database which consisted of about 500 mathematical equations from Clausius's 16 important papers (1850–1865) compiled in his two publications (1864 & 1867).[4] Having looked over these equations, several significant facts were discovered. The most important one was Clausius's approach for treating the first and (pre-) second laws of thermodynamics as a related set of equations, and it was noticed through a quick glance at our database. Through this approach Clausius proposed the form of entropy dS (dQ/T) as a complete differential for the reversible process corresponding to the (internal) energy dU of the first law, which was already considered a complete differential in the field of mechanics. Here look at the Figure 1 of the chronological table of development of Clausius (1822–1888),[5] the final mathematical equation of the second law of thermodynamics using the term entropy, which included an unequal sign for the irreversible process and which was presented by Clausius together with the word expression in 1865. However, the pre-second law started to be developed in 1850 when Clausius modified Carnot's material theory of heat to the mechanical theory of heat through simply adopting Carnot's idea that the temperature difference produced the motive power. There were several variations in progress while developing the pre-second law between 1850 and 1865. Detailed discussions were published in our paper.[6]

1.3. Experimental Data Table Analysis

Clausius was interested in experimental data although he did not carry out experiments himself. These experimental data were by E. Clapeyron, J. P. Joule, W. Thomson, and H. V. Regnault. We found three related experimental tables by Clapeyron (1834), W. Thomson (read 1849) and Clausius (1850). These tables include the so-called Carnot function C (the function of temperature) for 4 different temperatures, including 100 (Cent Gr.). The function C played an important role in the formation of the second law of thermody-

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[3] Eri Yagi and Haruo Hayashi, "Clausius's first and second laws of thermodynamics with Fourier's Influence," *Proceedings of the 20th International Congress of the History of Science*, Liege, 1997, Vol. 14, (Belgium, Breopls), pp. 132–141. Also in Ref 1. pp. 81–98.

[4] R. Clausius. *Abhandlungen über die Mechanische Wärmetheorie*, Erste Abtheilung Abhandlungen I–VIII (1864): Zweite IX–XVI (1867): (Freidrich Vieweg und Sohn)

[5] Fig 1 is also shown in Ref 1 as Figure 0–1 on p, 30.

[6] Eri Yagi and Rika Tadokoro, "Studies on the History of Thermodynamics through a Database," delivered at the British Society for the History of Mathematics Christmas Meeting, Birkbeck College, London, 17th December1998, *BSHM Newsletter*, No. 39 (1999): 8–9; also published in *Keizai-ronshu (Economic Review of Toyo University)*, 27 (2002): 299–310. Also in Ref. 1, pp. 99–112.

namics. Detailed discussions with these tables were published in our paper.[7]

2. Technical Term Analysis

2.1. Technical Terms in thermodynamics

We picked up 6 special technical terms, used in Clausius's 4 papers in thermodynamics through the traditional text analysis method. These papers were "Ueber die bewegende Kraft der Wärme und Gesetze, welche sich daraus für die Wärmelehre selbust ableiten lassen" (1850)[8], "Ueber eine veränderet Form des zweiten Hauptsatzes der mechanischen Wäermetheorie"(1854)[9], "Ueber die Anwendung des Satzes von der Aequivalentz der Verwandlungen auf die inner Arbeit"(1862)[10], and "Ueber die verschiedene für die Anwendung bequeme Formen der Hauptgleichungen der mechanischen Wäermetheorie"(1865).[11]

These 6 technical terms are "Aequivalent (Equivalent)", "Nicht umkehrbar (Nonreversible)", "Verwandlung (Transformation)", "Uncompensirte Verwandlung (Noncompensated transformation)", "Disgregation (Degree of dispersion)", and "Entropie (Entropy). The appearance of these terms is shown in this paper by Table 1 with Figure 1 of the chronological table of development by R. Clausius (1822–1888), which was taken from Figure 1 in Ref. 1.

The term Aequivalent (Equivalent) was used firstly in Clausius's paper of 1850 to express the principle of the equivalence between heat and work, namely, the 1st law of thermodynamics. Here Clausius wrote that his constant "A" expressed the equivalent of heat for the unit of work. It is worth pointing out that A is reciprocal to Joule's "J" which expressed the equivalent of work for the unit of heat. (Note that Clausius's equations of

Table 1. Clausius's Technical Terms in Thermodynamics

Clausius's Technical Terms	(1850–1865) Appearance: O			
Published years	1850	1854	1862	1865
Aequivalent	O	O	O	O
Verwandlung	X	O	O	O
Nicht umkehrbar	X	O	O	O
Umcompensirte Verwandlung	X	O	O	O
Disgregation	X	X	O	O
Entropie	X	X	X	O

[7] Eri Yagi, Rika Tadokoro and Haruo Hayashi, "Studies on Clausius various methods," (2002) in Ref. 1, pp. 13–37.

[8] R. Clausius, *Ann. d. Phys.* Vol. 79 (1850): 368–397 & 500–524. Abhandlung I in Ref. 4, pp. 16–78. *Phil.Mag.* Vol. (4) 2 (1850): 1–21 & 102–119.

[9] R. Clausius, *Ann. d. Phys.* Vol. 93 (1854): 481–506. Abhandlung IV in Ref. 4, pp. 127–154. *Phil. Mag.* Vol. (4) 12 (1854): 81–93.

[10] R. Clausius, *Ann. d. Phys.* Vol. 116 (1862): 73–112. Abhandlung VI in Ref. 4, pp. 242–279. *Phil. Mag.* Vol. (4) 24 (1862): 81–97 & 201–213.

[11] R. Clausius,*Ann.d.Phys.* Vol. 125 (1865): 353–400. Abhandlung IX in Ref. 4, Zweite pp. 1–44. R. Clausius (ed. by T. A. Hirst), *The Mechanical Theory of Heat* (London: Tyler and Francis, 1867): 327–365.

Carnot (1824): Motive power of fire.

Clapeyron (1834): Analytical expressions of Carnot (Q :heat, R :gas constant, p :pressure, C : Carnot's temperature function, v :volume):

$$Q - Q' = RC\log\frac{p'}{p} = RC\log\frac{v}{v'}$$

Clausius (1847): Light absorbtion in the atmosphere.

Helmholtz (1847): Conservation of "Force" (mechanical energy):

$$\frac{1}{2}mQ^2 - \frac{1}{2}mq^2 = -\int_r^R \varphi dr$$

Joule (1843-1850): Experimental determination of the mechanical equivalent of heat, J

W.Thomson (1848): Absolute thermometric scale.

W.Thomson (1849): The first law of thermodynamics (W: work, t :tempetature, $\mu = \frac{1}{C}$)

$$\frac{W}{Q} = \frac{\mu(1-Et)}{E} \quad (=J,\ 1851)$$

where μ denotes Carnot's coefficient, E expansion coefficient.

Clausius (1850): The first law of thermodynamics

$$dQ = dU + A \cdot R\frac{a+t}{v}dv \qquad (a+t=T,\ 1854)$$

where dU indicates the energy of a gas.

Clausius (1854): $\int \frac{dQ}{T} = 0$ (reversible cyclical process)

Clausius (1862): $\int \frac{dQ+dH}{T} + \int dZ \geqq 0$

Z :"disgregation", H : heat of the body.

Clausius (1865): "energy of the body" : $dU = dQ - dw$, or $dQ = dU + dw$.

"entropy" : $dS = \frac{dQ}{T}$ (reversible process),

or $\int \frac{dQ}{T} \leqq 0$ (reversible & irreversible processes).

1. The energy of the universe is constant.

2. The entropy of the universe tends to a maximum.

Figure 1. Chronological Table of Development by R. Clausius (1822–1888) and People before him and his contemporaries, where the historical background of the mechanical theory of heat is shown.

thermodynamics were written in the heat unit while Joule's were in the work unit). In his paper of 1854, Clausius expanded the usage of the term Aequivalent (Equivalent) together with the term Verwandlung (Transformation). Here he named the transformation from work to heat which was assumed to have the positive equivalent value as the first kind of transformation. In addition to that, the transformation of heat from higher to lower temperatures, also the positive equivalent value, was named as the second kind of transformation. Clausius tried to find the common equivalent value for the above two kinds of transformation. Then he succeeded in finding the form of value as Q/T (heat/temperature) which is the mathematical form of Carnot's law (based on the material theory of heat) for the reversible process. The term Uncompensirte Verwandlung (Noncompensated transformation) indicated Clausius's interest in the irreversible process, which started from 1854. The term Disgregation (Degree of dispersion) was used by Clausius from 1862 to show the micro nature of working substances. He mentioned that heat increased the Disgregation of molecules. This expression meant the micro nature of thermodynamics from the beginning before the appearance of the term Entropie (Entropy).

2.2. Technical Term Analysis in the theory of electricity

2.2.1. Applying the method of text mining to the analysis on Clausius's papers

We started to investigate Clausius's four papers (1852–57) on the theory of electricity through the traditional text analysis with the help of our database. Here the first and (pre-) second laws of thermodynamics were applied to solve electrical phenomena. Further, the way of approaching small particles of matter was developed. In addition to the above, we began to perform our preliminary studies on the technical term analysis to his 12th paper.[12] Now we will describe how we applied "noun analysis" on Clausius's 12th paper ("Abhandlung XII" in German[13]). Here we used some basic methods of text mining.

2.2.2. What is "text mining"?

Text mining is a method of document analysis, which mainly uses computational and statistical procedures and tools. According to the definition of text mining by Feldman and Sanger;

> Text mining can be broadly defined as a knowledge-intensive process in which a user interacts with a document collection over time by using a suite of analysis tools.[14]

Text mining is often referred to as a part of data mining, machine learning, and com-

[12] Eri Yagi and Rika Tadokoro, "Theory of electricity by R. Clausius in the development of Thermodynamics," Bicentenary of Invention of the Battery by A.Volta, Volta and the History of Electricity Conference, Como & Pavia, 11–15 September, 1999, *Book of Abstracts* (1999), p. 29, *Volta and the History of Electricity,* ed. by Fabio B. & E.Gionnetto, pp. 299–307 (Milan: Hoepli International Publisher, 2003). Also in Ref. 1, pp. 21–22 and pp. 113–123.

[13] R. Clausius, "Ueber die Anwendung der mechanischen Wärmetheorie auf die thermoelectrischen Erscheinungen," *Ann. d. Phys.* Vol. 90 (1853): 513–544. Abhandlung XII in Ref. 4, Zweite pp. 175–201.

[14] R. Feldman and J. Sanger, *The Text Mining Handbook, Advanced Approaches in Analyzing Unstructured Data* (Cambridge University Press, 2007), p. 1

Table 2. Appearance Frequency (Top 20) of the Technical Terms.

(unit: times)

Words (German)	Appearance Frequency	Meaning (English)
Wärme	69	heat
Differenz	59	difference
Arbeit	55	work
Strom	46	electric current
Kraft	44	power, strength, energy
Stoff	41	substance
Temperatur	35	Temperature
Beruhrungsstelle	33	(thermopile) Junction
Electricitat	30	electricity
Metall	30	metal
Theil	30	part
Ausdruck	23	expression
Kette	21	(thermopile) chain
Gleichung	20	equation
Leiter	20	conductor
Werth	20	value
Annahme	19	hypothesis
Erscheinung	19	phenomenon
Gesetz	19	law
Schlus	19	conclusion

putational linguistics, because basically this method requires such retrieval capability to check the data quantity and quality. It generally requires "machine readable" data and some tools to cleanse objective data, to apply the statistical computation and to draw some graphs.

In this paper, we would like to focus on computational and statistical text mining as a method of text analysis to analyze again Clausius's 12th paper.

2.2.3. Previous text analysis on "Abhandlung XII"

As mentioned above, we have already tried the method of technical term analysis on Clausius's 12th paper (called Abhandlung XII in German).[15]

In our previous paper it was realized that the thermoelectric current was an important applicable (appropriate) phenomenon on both the first and pre-second laws of thermodynamics. Further, the distribution graph of frequently appeared physicists in the paper was interestingly classified into such two fields of people as thermodynamics and electricity.[16]

[15] Eri Yagi, Rika Tadokoro and Haruo Hayashi, "Studies on R. Clausius through various useful methods," in Ref. 1, pp. 21–22.

[16] See Figure IV-I in Ref. 1, p. 37.

Table 3. Appearance Frequency of the Technical Terms' Categories (sub-fields in Physics) in Clausius's 12th Paper.

(unit: times)

Category	Appearance Frequency
physics	375
electromagnetism	247
thermodynamics	158
substance	102
person's name	39
place name	4
miscellaneous	803

2.2.4. Applying the method of text mining to "Abhandlung XII"

We applied the method to "Abhandlung XII" again along the following four processes:

(1) scanning the paper with a scanner,

(2) turning the scanned image into text data through OCR (Optical Character Recognition) software, or input the text data as a computer-readable format by hand,

(3) cleansing the data so it is readable by a computer,

(4) making a dictionary for the text analysis.

Through these procedures, each process itself is very simple but it is time-consuming and increases the size of each document as well as the number of documents. There is some software that can automatically handle the processes (1) to (3). However, cleansing the data (which means eliminating OCR errors, formatting the text, and correcting words) manually is indispensable for analyzing a document, and also making the dictionary for each purpose by hand.

So we have firstly decided to carry out the above procedures for Abhadlung XII, because of its word quantity, which is of a relatively large volume including the same kinds of words that appear in any other Clausius paper. For these procedures, we aim to build our own method of technical term analysis.

2.2.5. Analyzing the Result

The following tables and figures were made as a result of our studies:

Table 2. Appearance Frequency (Top 20) of the Technical Terms.

Table 3. Appearance Frequency of the Technical Terms' Categories (sub-fields in Physics) in Clausius's 12th Paper. Note: Each category indicated in this table is selected from a proper sub-field in Physics. Each noun in Abhandlung XII is classified by one of this category. Each word, which appears here, has two or more meanings but we selected only one of the meanings.

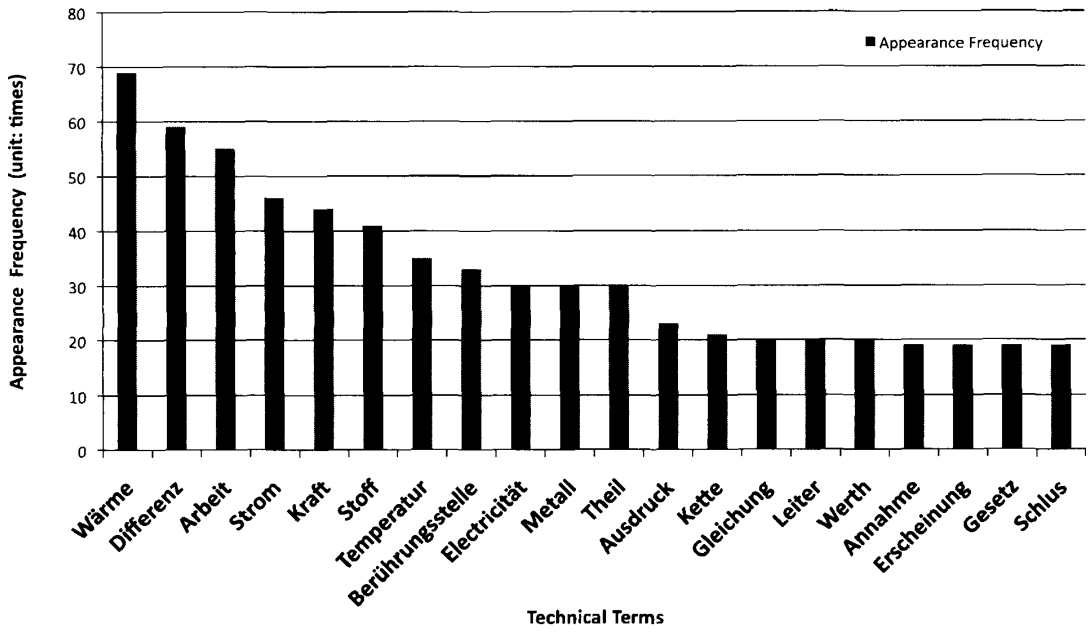

Figure 2. Appearance Frequency graph of Technical Terms from Table 2.

2.2.6. Discussion

Through the above studies by the text mining, where we used typical nouns as technical terms, the essential character of Clausius's 12th paper became clearer, e.g. the top 20 terms of the high appearance frequency are mainly observed (see Table 3) in such two sub-fields as thermodynamics and electromagnetism. Here let us discuss the first top three technical terms, namely. "Wärm(heat)", "Differenz(difference)," and "Arbeit(work)." The term "Wärm(heat)" is the most important one among Clausius's papers (in 1850, 1854, and 1862) of thermodynamics where dQ or Q(heat) was written on the left hand side of his mathematical equations to answer the question of what heat is in the first law of thermodynamics. This was already pointed out through our mathematical equation analysis that nicely coincides with the result through our text mining. The term "Differenz(difference)" should have been handled with its adjective "electrische(electric)" as one unit of "electrische Differenz" but the term "Differenz(difference)" was simply classified to the 'miscellaneous' category through our analysis on this occasion. Therefore, we realized that there are some difficulties, namely, how to classify each noun or each unit to a proper category. Fortunately, having traced the term "electrische Differenz," we found that Clausius himself changed it to "Potentialniveaudifferentz" in 1866, which is currently called "Potentialdifferenz" as one term in German. The term "Arbeit(work)" belonged to the category of physics in the field of mechanics. Naturally, the term "Arbeit(work)" had one of the most important roles in the 1st law of thermodynamics since Clausius himself mentioned the law in his paper (1854) as the theorem of equivalence of heat and work. There were five terms classified to the category of electromagnetism among the top 20 terms. Figure 3

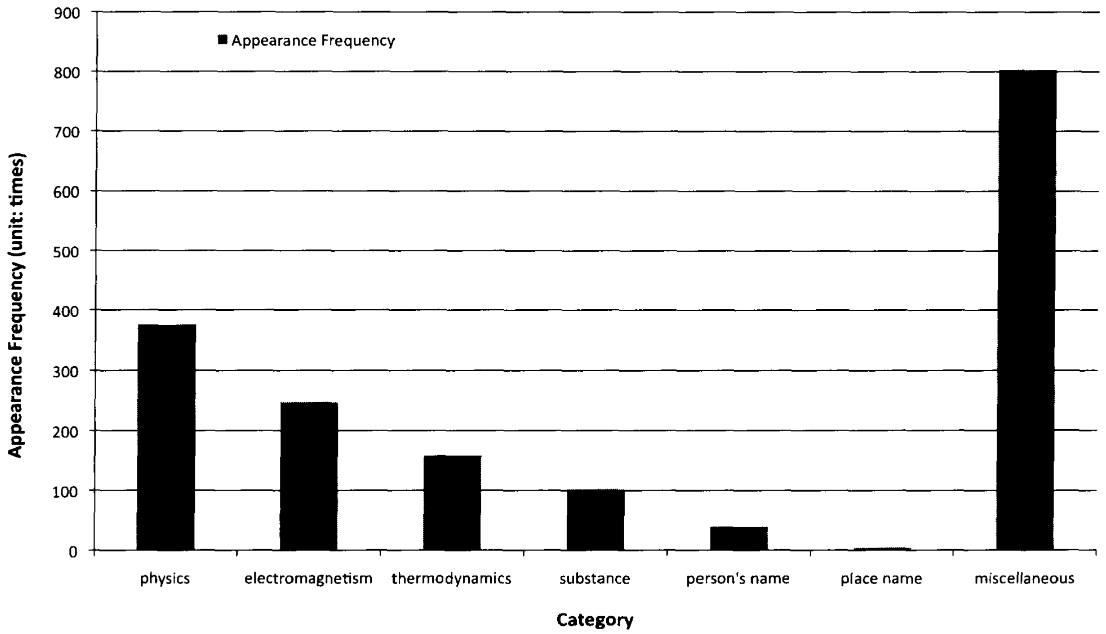

Figure 3. Appearance Frequency graph of the Technical Terms' Categories from Table 3.

indicated clearly the close connection between thermodynamics and electromagnetism although some terms in the 'miscellaneous' category might be changed to electromagnetism with the consideration of their adjectives.

3. Remarks and Conclusions

We are very much interested in developing the various methods to study the history of physics (science) together with studies on the history of the mechanical theory of heat, focusing on R. Clausius. On this occasion the results of our studies through the four various methods, especially those through the method of technical term analysis were mainly described. Our technical term analysis has two approaches to the original texts: firstly picking up the technical terms through the traditional method with the help of our database, and secondly doing them through the appearance frequency with the help of a computer. The second approach, e.g., the text mining has not been applied to the studies of history of physics (science) although it has been used among humanities studies (identifying authors of literature), social investigations (through questionnaires), and medical fields. However, the above two approaches to our technical term analysis played important roles in expanding and strengthening our research results rather than only relying on the traditional text analysis. Non-native Germans, like us, could not study Clausius's classical research papers (with the manuscripts) of the 19th century so easily because they present certain difficulties even for current German researchers. This is why we are very much interested in developing the various methods to study the history of physics (science).

Acknowledgements

This paper is mainly based on our paper, delivered in the afternoon of the 29th July, 2009, Session 9.2 at the 23rd International Congress of the History of Science, Budapest, Hungary.[17] We deeply appreciate to the supportive participants at the meeting. Thanks also go to the Editor, Professor Takehiko Hashimoto's interest in our delivered paper and encouragement for the submission of the paper to this journal. Our studies on Clausius have been strongly encouraged by the Clausius Tower (Project) Society at Koszalin University of Technology, Koszalin, Poland, to which Yagi was invited for the solemn session of the ceremony for the setting of the foundation cornerstone for the new Clausius Tower on September 21st 2009.[18]

(Received on 15 March 2010; Accepted on 7 May 2010)

[17] E. Yagi & R. T. Okamoto, "A History of Entropy through various Methods," *Abstract*, the 23rd International Congress of the History of Science, Budapest, Hungary (2009). See also Professor Emeritus Yagi's Blog: http://www.schaft.org/wp/

[18] E. Yagi, "R. Clausius's contribution to the first and second laws of thermodynamics," *Book of Abstract, 4th Symposium on Vacuum based Science & Technology*, September, Koszalin, Poland (2009) p. C1. The meaning of the session was described on Professor Emeritus Yagi's Blog: http://www.schaft.org/wp/ Also visit the website of the Clausius Society: http://www.clausius-tower-society.koszalin.pl/